

# The Ellipsoidal Universe in the Planck Satellite Era

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## ABSTRACT

Recent Planck data confirm that the cosmic microwave background displays the quadrupole power suppression together with large scale anomalies. Progressing from previous results, that focused on the quadrupole anomaly, we strengthen the proposal that the slightly anisotropic ellipsoidal universe may account for these anomalies. We solved at large scales the Boltzmann equation for the photon distribution functions by taking into account both the effects of the inflation produced primordial scalar perturbations and the anisotropy of the geometry in the ellipsoidal universe. We showed that the low quadrupole temperature correlations allowed us to fix the eccentricity at decoupling,  $e_{\text{dec}} = (0.86 \pm 0.14) 10^{-2}$ , and to constraint the direction of the symmetry axis. We found that the anisotropy of the geometry of the universe contributes only to the large scale temperature anisotropies without affecting the higher multipoles of the angular power spectrum. Moreover, we showed that the ellipsoidal geometry of the universe induces sizable polarization signal at large scales without invoking the reionization scenario. We explicitly evaluated the quadrupole TE and EE correlations. We found an average large scale polarization  $\Delta T_{\text{pol}} = (1.20 \pm 0.38) \mu\text{K}$ . We point out that great care is needed in the experimental determination of the large-scale polarization correlations since the average temperature polarization could be misinterpreted as foreground emission leading, thereby, to a considerable underestimate of the cosmic microwave background polarization signal.

**Key words:** cosmic microwave radiation - cosmology: theory.

## 1 INTRODUCTION

The Cosmic Microwave Background (CMB) anisotropy data produced by the final analysis of the Wilkinson Microwave Anisotropy Probe (WMAP) (Bennett et al. 2013; Hinshaw et al. 2013) and, more recently, by the Planck satellite (Ade et al. 2013a,c,d) confirm the standard cosmological Lambda Cold Matter ( $\Lambda$ CDM) model at an unprecedented level of accuracy. At large scales, however, several anomalous features have been reported: an unusual alignment of the preferred axes of the quadrupole and octopole (Land & Magueijo 2005; de Oliveira-Costa et al. 2004; Ralston & Jain 2004; Copi et al. 2006), non-Gaussian signatures due to a cold spot (Cruz et al. 2005), an hemispherical power asymmetry at large scales (Eriksen et al. 2004; Hansen, Banday & Gorski 2004). Nevertheless, we feel that one of the most important discrepancy resides in the low quadrupole moment, which signals an important suppression of power at large scales. In fact, the Planck collaboration reported a statistical significant tension between the best fit  $\Lambda$ CDM model and the large-scale spectrum due to a systematic lack of power for  $\ell \lesssim 40$  (Ade et al. 2013c) and to anomalies in the statistical isotropy of the sky maps (Ade et al. 2013e). If these anomalies should turn out to have a cosmological origin, then it could have far reaching consequences for our present understanding of the universe.

Quite recently it has been suggested (Campanelli, Cea & Tedesco 2006, 2007; Cea 2010) that, if one admits that the large-scale spatial geometry of our universe is only plane-symmetric with eccentricity at decoupling of order  $10^{-2}$ , then the quadrupole amplitude can be drastically reduced without affecting higher multipoles of the angular power spectrum of the temperature anisotropy. As discussed in Campanelli, Cea & Tedesco (2007), the anisotropic expansion described by a plane-symmetric metric can be generated by cosmological magnetic fields or topological defects, such as cosmic domain walls or

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cosmic strings. Indeed, topological cosmic defects are relic structures that are predicted to be produced in the course of symmetry breaking in the hot, early universe (e.g., see Vilenkin & Shellard (1994)).

### 1.1 Outline of the Results

In an isotropic and homogeneous universe the most general metric is the Friedmann-Robertson-Walker (FRW) metric (see, for instance, Peebles 1993). In particular, the metric of standard cosmological model is given by <sup>1</sup>:

$$ds^2 = -dt^2 + a^2(t)\delta_{ij} dx^i dx^j . \quad (1)$$

If we assume that the large-scale spatial geometry of our universe is only plane-symmetric, then the metric Eq. (1) is replaced with the ellipsoidal universe metric:

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij}) dx^i dx^j , \quad (2)$$

where  $h_{ij}$  is a metric perturbation which we assume to be of the form:

$$h_{ij} = -e^2(t) n_i n_j . \quad (3)$$

In Eq. (3)  $e(t) = \sqrt{1 - (b(t)/a(t))^2}$  is the ellipticity and the unit vector  $\vec{n}$  determines the direction of the symmetry axis.

In this paper we shall further elaborate on the ellipsoidal universe proposal and extend previous investigations in several directions. For reader's convenience, it is useful to summarize the main results of the present paper.

First, we consider the Boltzmann equation for the photon distribution in the ellipsoidal universe, discussed for the first time in Cea (2010), by taking into account also the effects of the cosmological inflation produced primordial scalar perturbations. In the large scale approximation we explicitly show that the CMB temperature fluctuations can be written as:

$$\Delta T \simeq \Delta T^I + \Delta T^A , \quad (4)$$

where  $\Delta T^I$  and  $\Delta T^A$  are the temperature fluctuations induced by the cosmological scalar perturbations and by the spatial anisotropy of the metric of the universe, respectively. Since the temperature anisotropies caused by the inflation produced primordial scalar perturbations are discussed in several textbooks (Dodelson 2003; Mukhanov 2005), we focus on the temperature fluctuations induced by the anisotropy of the metric by solving the relevant Boltzmann equation. At large scales we solve that equation and determine the solutions relevant for the CMB temperature and polarization fluctuations. Indeed, it is well known (Rees 1968; Negroponte & Silk 1980; Basko & Polnarev 1980) that anisotropic cosmological models give sizable contributions to the large scale polarization of the cosmic microwave background radiation. In fact, polarization measurements could provide a unique signature of cosmological anisotropies.

We go beyond the approximations adopted in Cea (2010) and confirm that the main contributions to the CMB temperature fluctuations affect the quadrupole correlations. In addition, we also show that the effects of the spatial anisotropy of the metric of the universe extend to low-lying multipoles  $\ell \sim 10$ .

As is well known, the CMB temperature fluctuations are fully characterized by the power spectrum:

$$(\Delta T_\ell)^2 \equiv \mathcal{D}_\ell = \frac{\ell(\ell+1)}{2\pi} C_\ell , \quad C_\ell = \frac{1}{2\ell+1} \sum_{m=-\ell}^{+\ell} |a_{\ell m}|^2 . \quad (5)$$

In particular, the quadrupole anisotropy refers to the multipole  $\ell = 2$ . Remarkably, the Planck data (Ade et al. 2013c) confirmed that the observed quadrupole anisotropy:

$$(\Delta T_2)^2 = \mathcal{D}_2 \simeq 299.5 \mu K^2 , \quad (6)$$

is much smaller than the quadrupole anisotropy expected according to the best fit  $\Lambda$ CDM model to the Planck data:

$$(\Delta T_2^I)^2 = 1150 \pm 727 \mu K^2 . \quad (7)$$

Note that in Eq. (6) we are neglecting the rather small measurement errors, while the uncertainties due to the so-called cosmic variance are included in the theoretical expectations, Eq. (7). In fact, using Eq. (4) we show that the quadrupole temperature anisotropy can be reconciled with observations in the ellipsoidal universe if the eccentricity at decoupling is:

$$e_{\text{dec}} = (0.86 \pm 0.14) 10^{-2} , \quad (8)$$

irrespective of the physical mechanism responsible for the generation of the spatial anisotropy in the early universe. Moreover, if we denote with  $b_n$  and  $l_n$  the galactic latitude and longitude of the symmetry axis respectively, we also were able to show that the axis of symmetry were constrained to:

<sup>1</sup> Note that through the paper we shall use units in which  $c = 1$ ,  $\hbar = 1$  and  $k_B = 1$ .

$$b_n \simeq \pm 17^\circ, \quad (9)$$

while the longitude  $b_n$  turns out to be poorly constrained, in qualitative agreement with Campanelli, Cea & Tedesco (2007). As concern the CMB polarization, we confirm our previous result (Cea 2010) that the ellipsoidal geometry of the universe induces sizable polarization signal at large scales without invoking the reionization scenario. In particular, we find an average large scale polarization:

$$\Delta T_{pol} \equiv \frac{1}{4\pi} \int d\Omega \Delta T^E(\theta, \phi) = (1.20 \pm 0.38) \mu K, \quad (10)$$

where  $\Delta T^E(\theta, \phi)$  is the polarization of the CMB temperature fluctuations. Moreover, we evaluate the quadrupole temperature-polarization cross-correlation (TE) and polarization-polarization (EE) correlation. We find:

$$\Delta T_2^{TE} = 3.14 \pm 0.76 \mu K, \quad (11)$$

and

$$\Delta T_2^{EE} = 0.83 \pm 0.27 \mu K. \quad (12)$$

These values should be compared with the available observational data. Since the Planck collaboration does not yet make public the large scale polarization data, we must rely on the final analysis of the Wilkinson Microwave Anisotropy Probe collaboration. The WMAP nine-year full-sky maps of the polarization detected at large scales in the foreground corrected maps an average E-mode polarization power (Bennett et al. 2013; Hinshaw et al. 2013). In particular for the quadrupole correlations we have (including only the statistical uncertainties):

$$\frac{l(l+1)}{2\pi} C_{l=2}^{TE} = 2.4439 \pm 2.2831 \mu K^2, \quad WMAP \text{ nine - years} \quad (13)$$

and

$$\frac{l(l+1)}{2\pi} C_{l=2}^{EE} = -0.0860 \pm 0.0247 \mu K^2, \quad WMAP \text{ nine - years} \quad (14)$$

Using the definition in Eq. (5) we estimate:

$$\Delta T_2^{TE} = 1.56 \pm 0.73 \mu K, \quad WMAP \text{ nine - years} \quad (15)$$

which within two standard deviations agrees with our result Eq. (11). On the other hand, as concern the quadrupole EE correlation, Eq. (14) at best gives an upper bound which, however, is not consistent with our result Eq. (12). We believe that this discrepancy could be due to the fact that in the ellipsoidal universe model, at variance of the standard reionization scenario, there is a non-zero average temperature polarization. In fact, the eventual presence of an average temperature polarization could be misinterpreted as foreground emission leading to an underestimate of the cosmic microwave background polarization signal.

The plan of the paper is as follows. In sect. 2 we discuss the Boltzmann equation of the cosmic background radiation in the ellipsoidal universe. In sect. 3 we determine the solutions of the Boltzmann equation at large scales. Sect. 4 is devoted to the problem of the quadrupole anomaly in the temperature-temperature fluctuation correlations. In sect. 5 we discuss the large scale polarization. In particular, we determine the quadrupole TE and EE correlations. Finally, our conclusions are drawn in sect. 6. Some technical details are relegated in appendix A, while in appendix B we discuss the multipole expansion of the large scale temperature anisotropies.

## 2 THE BOLTZMANN EQUATION IN THE ELLIPSOIDAL UNIVERSE

We are interested in the temperature fluctuations of the cosmic background radiation induced by eccentricity of the universe and by the inflation produced primordial cosmological perturbations. We assume that the photon distribution function  $f(\vec{x}, t)$  is an isotropically radiating blackbody at a sufficiently early epoch. The subsequent evolution of  $f(\vec{x}, t)$  is determined by the Boltzmann equation (Dodelson 2003; Mukhanov 2005):

$$\frac{df}{dt} = \left( \frac{\partial f}{\partial t} \right)_{coll}, \quad (16)$$

where  $(\frac{\partial f}{\partial t})_{coll}$  is the collision integral which takes care of Thomson scatterings between matter and radiation.

The metric of the standard FRW universe is:

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j. \quad (17)$$

Here, we are interested in primordial scalar perturbations induced by the inflation. In the conformal Newtonian, or longitudinal gauge (Mukhanov 2005), the metric Eq. (17) can be written as:

$$ds^2 = -[1 + 2\Psi(\vec{x}, t)] dt^2 + a^2(t) \delta_{ij} [1 + 2\Phi(\vec{x}, t)] dx^i dx^j . \quad (18)$$

In this gauge the perturbations to the metric are determined by the functions  $\Psi(\vec{x}, t)$  and  $\Phi(\vec{x}, t)$  which correspond to the Newtonian potential and the perturbation to the spatial curvature, respectively. In the ellipsoidal universe the metric would be:

$$ds^2 = -[1 + 2\Psi(\vec{x}, t)] dt^2 + a^2(t) [\delta_{ij} + h_{ij}] [1 + 2\Phi(\vec{x}, t)] dx^i dx^j , \quad (19)$$

where  $h_{ij}$  is given by Eq. (3). However, both the primordial perturbations  $\Psi(\vec{x}, t)$ ,  $\Phi(\vec{x}, t)$  and the ellipticity are to be considered small at the times and scales of interest. Therefore in the following we shall neglect all terms quadratic in them. Accordingly, instead of Eq. (19) we have:

$$ds^2 = -[1 + 2\Psi(\vec{x}, t)] dt^2 + a^2(t) \{ \delta_{ij} [1 + 2\Phi(\vec{x}, t)] + h_{ij} \} dx^i dx^j . \quad (20)$$

We are interested in the anisotropies in the cosmic distribution of photons. To this end, we need to evaluate the photon distribution function  $f(\vec{x}, t)$  which satisfies the Boltzmann equation Eq. (16). Actually, the distribution function depends on the space-time point  $x^\mu$  and the momentum vector  $p^\mu$  defined by:

$$p^\mu = \frac{dx^\mu}{d\lambda} , \quad (21)$$

where  $\lambda$  parametrizes the particle's path. For massless particles we, obviously, have:

$$P^2 \equiv g_{\mu\nu} p^\mu p^\nu = 0 . \quad (22)$$

Using the metric in Eq. (20) and defining:

$$p^2 \equiv g_{ij} p^i p^j , \quad (23)$$

from Eq. (22) we easily obtain:

$$p^0 \simeq p [1 - \Psi] . \quad (24)$$

It is convenient to consider the distribution function as a function of the magnitude of momentum  $p$  and momentum direction  $\hat{p}^i$ ,  $\delta_{ij} \hat{p}^i \hat{p}^j = 1$ . Therefore we have:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} \frac{dx^i}{dt} + \frac{\partial f}{\partial p} \frac{dp}{dt} + \frac{\partial f}{\partial \hat{p}^i} \frac{d\hat{p}^i}{dt} . \quad (25)$$

Now we note that:

$$\frac{dx^i}{dt} = \frac{dx^i}{d\lambda} \frac{d\lambda}{dt} = \frac{p^i}{p^0} . \quad (26)$$

Let us write

$$p^i = C \hat{p}^i , \quad (27)$$

then it is easy to find:

$$C \simeq \frac{p}{a(t)} [1 - \Phi - \frac{1}{2} h_{ij} p^i p^j] . \quad (28)$$

So that we have:

$$\frac{dx^i}{dt} \simeq \frac{\hat{p}^i}{a(t)} [1 - \Phi + \Psi - \frac{1}{2} h_{ij} p^i p^j] . \quad (29)$$

Thus, we get:

$$\frac{df}{dt} \simeq \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} \frac{\hat{p}^i}{a(t)} [1 - \Phi + \Psi - \frac{1}{2} h_{ij} p^i p^j] + \frac{\partial f}{\partial p} \frac{dp}{dt} + \frac{\partial f}{\partial \hat{p}^i} \frac{d\hat{p}^i}{dt} \simeq \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} \frac{\hat{p}^i}{a(t)} + \frac{\partial f}{\partial p} \frac{dp}{dt} \quad (30)$$

since  $\frac{\partial f}{\partial x^i}$  is already a first-order term. To evaluate  $\frac{dp}{dt}$ , we note that the time component of the geodesic equations gives:

$$\frac{dp^0}{d\lambda} = -\Gamma_{\alpha\beta}^0 p^\alpha p^\beta . \quad (31)$$

Since

$$\frac{dp^0}{d\lambda} = \frac{dp^0}{dt} \frac{dt}{d\lambda} = p^0 \frac{dp^0}{dt} , \quad (32)$$

after using Eq. (24) we obtain:

$$\frac{dp}{dt} \simeq p \frac{d\Psi}{dt} - \Gamma_{\alpha\beta}^0 \frac{p^\alpha p^\beta}{p} [1 + 2\Psi] = p \left[ \frac{\partial \Psi}{\partial t} + \frac{\hat{p}^i}{a(t)} \frac{\partial \Psi}{\partial x^i} \right] - \Gamma_{\alpha\beta}^0 \frac{p^\alpha p^\beta}{p} [1 + 2\Psi] , \quad (33)$$

Moreover, a standard calculation (Dodelson 2003) shows that:

$$\Gamma_{\alpha\beta}^0 \frac{p^\alpha p^\beta}{p} \simeq p(1 - 2\Psi) \left[ \frac{\partial\Psi}{\partial t} + 2\frac{\hat{p}^i}{a(t)} \frac{\partial\Psi}{\partial x^i} + \frac{\partial\Phi}{\partial t} + \frac{1}{2}\hat{p}^i \hat{p}^j \frac{\partial h_{ij}}{\partial t} + H \right], \quad (34)$$

where  $H = \dot{a}/a$  is the Hubble rate. Finally, inserting Eqs. (33) and (34) into Eq. (30) and collecting terms we obtain:

$$\frac{df}{dt} \simeq \frac{\partial f}{\partial t} + \frac{\hat{p}^i}{a(t)} \frac{\partial f}{\partial x^i} - p \frac{\partial f}{\partial p} \left[ H(t) + \frac{\partial\Phi}{\partial t} + \frac{\hat{p}^i}{a(t)} \frac{\partial\Psi}{\partial x^i} + \frac{1}{2}\hat{p}^i \hat{p}^j \frac{\partial h_{ij}}{\partial t} \right]. \quad (35)$$

To go further we expand the photon distribution about its zero-order Bose-Einstein value:

$$f_0(p, t) = \frac{1}{e^{\frac{p}{T(t)}} - 1}. \quad (36)$$

We write:

$$f(\vec{x}, t, p, \hat{p}) = \frac{1}{e^{\frac{p}{T(t)[1 + \Theta(\vec{x}, t, p, \hat{p})]}} - 1}, \quad (37)$$

and expand to the first order in the perturbation  $\Theta(\vec{x}, t, p, \hat{p})$ :

$$f(\vec{x}, t, p, \hat{p}) \simeq f_0(p, t) - p \frac{\partial f_0}{\partial p} \Theta(\vec{x}, t, p, \hat{p}). \quad (38)$$

Using the relation  $\frac{\partial \ln f_0}{\partial \ln p} \simeq -1$  which is valid in the Rayleigh-Jeans region, we can rewrite Eq. (38) as:

$$f(\vec{x}, t, p, \hat{p}) \simeq f_0(p, t) [1 + \Theta(\vec{x}, t, p, \hat{p})]. \quad (39)$$

If we neglect the perturbations, it is easy to see that the zero-order Boltzmann equation is satisfied by the Planck distribution Eq. (36) with  $T(t) \sim \frac{1}{a(t)}$ . To determine the perturbed distribution  $\Theta(\vec{x}, t, p, \hat{p})$  we need to evaluate the Boltzmann equation to the first order. From Eqs. (35) and (37) it follows that:

$$\left( \frac{df}{dt} \right)_{first\ order} \simeq -p \frac{\partial f_0}{\partial p} \left\{ \frac{\partial\Theta}{\partial t} + \frac{\hat{p}^i}{a(t)} \frac{\partial\Theta}{\partial x^i} + \frac{\partial\Phi}{\partial t} + \frac{\hat{p}^i}{a(t)} \frac{\partial\Psi}{\partial x^i} + \frac{1}{2}\hat{p}^i \hat{p}^j \frac{\partial h_{ij}}{\partial t} \right\}. \quad (40)$$

Thus the first-order Boltzmann equation becomes:

$$\frac{\partial\Theta}{\partial t} + \frac{\hat{p}^i}{a(t)} \frac{\partial\Theta}{\partial x^i} + \frac{\partial\Phi}{\partial t} + \frac{\hat{p}^i}{a(t)} \frac{\partial\Psi}{\partial x^i} + \frac{1}{2}\hat{p}^i \hat{p}^j \frac{\partial h_{ij}}{\partial t} \simeq \frac{1}{f_0} \left( \frac{\partial f}{\partial t} \right)_{coll}. \quad (41)$$

The collision integral is in general a non linear functional of the distribution function. However, in the first order approximation it is a linear functional of  $\Theta(\vec{x}, t, p, \hat{p})$ . Moreover, since we are interested in the solutions of the Boltzmann equation at large scales, we may neglect the effects due to the bulk velocity of the electrons which participate to the photon Compton scatterings. In this case the collision integral can be considered a linear homogeneous functional of the distribution function  $\Theta(\vec{x}, t, p, \hat{p})$ . As a consequence, if we write

$$\Theta(\vec{x}, t, p, \hat{p}) \simeq \Theta^A(\vec{x}, t, p, \hat{p}) + \Theta^I(\vec{x}, t, p, \hat{p}), \quad (42)$$

then we also have:

$$\left( \frac{\partial f}{\partial t} \right)_{coll} [\Theta] \simeq \left( \frac{\partial f}{\partial t} \right)_{coll} [\Theta^A] + \left( \frac{\partial f}{\partial t} \right)_{coll} [\Theta^I]. \quad (43)$$

In fact, Eq. (41) suggests that we may associate  $\Theta^A$  and  $\Theta^I$  with the temperature fluctuations induced by the spatial anisotropy of the geometry of the universe and by the scalar perturbations generated during the inflation, respectively. Accordingly we set:

$$\frac{\partial\Theta^I}{\partial t} + \frac{\hat{p}^i}{a(t)} \frac{\partial\Theta^I}{\partial x^i} + \frac{\partial\Phi}{\partial t} + \frac{\hat{p}^i}{a(t)} \frac{\partial\Psi}{\partial x^i} \simeq \frac{1}{f_0} \left( \frac{\partial f}{\partial t} \right)_{coll} [\Theta^I], \quad (44)$$

and

$$\frac{\partial\Theta^A}{\partial t} + \frac{\hat{p}^i}{a(t)} \frac{\partial\Theta^A}{\partial x^i} + \frac{1}{2}\hat{p}^i \hat{p}^j \frac{\partial h_{ij}}{\partial t} \simeq \frac{1}{f_0} \left( \frac{\partial f}{\partial t} \right)_{coll} [\Theta^A]. \quad (45)$$

We note that from Eq. (37) it follows that the distribution function  $\Theta(\vec{x}, t, p, \hat{p})$  is identified with the temperature contrast function:

$$\Theta(\vec{x}, t, p, \hat{p}) = \frac{\Delta T(\vec{x}, t, p, \hat{p})}{T(t)}. \quad (46)$$

Therefore, at large scales Eq. (42) implies that:

$$\Delta T(\vec{x}, t, p, \hat{p}) \simeq \Delta T^A(\vec{x}, t, p, \hat{p}) + \Delta T^I(\vec{x}, t, p, \hat{p}). \quad (47)$$

Note that Eq. (47) was assumed in Campanelli, Cea & Tedesco (2006, 2007); Cea (2010). Even though this hypothesis was considered reasonable, an explicit proof was lacking. Our discussion shows that Eq. (47) arises as a natural consequence of the Boltzmann equation which, however, is valid only at large distances.

We may conclude that to determine the CMB temperature fluctuations at large scales we need to solve the Boltzmann equations Eqs. (44) and (45). Eq. (44) is the Boltzmann equation of the standard  $\Lambda$ CDM cosmological model, and it has been extensively discussed in several textbooks (Dodelson 2003; Mukhanov 2005). Therefore, in the following we focus on the Boltzmann equation Eq. (45), derived for the first time in Cea (2010), which allows us to find the CMB temperature fluctuations caused by the anisotropy of the geometry of the universe.

### 3 LARGE SCALE SOLUTIONS OF THE BOLTZMANN EQUATION

In this section we discuss the Boltzmann equation in the ellipsoidal universe Eq. (45):

$$\frac{\partial \Theta(\vec{x}, t, p, \hat{p})}{\partial t} + \frac{\hat{p}^i}{a(t)} \frac{\partial \Theta(\vec{x}, t, p, \hat{p})}{\partial x^i} + \frac{1}{2} \hat{p}^i \hat{p}^j \frac{\partial h_{ij}}{\partial t} \simeq \frac{1}{f_0} \left( \frac{\partial f}{\partial t} \right)_{coll} . \quad (48)$$

where, for simplicity, the superscript A has been dropped. In Cea (2010) we have discussed the solutions of Eq. (48) by neglecting the spatial dependence of the temperature contrast function  $\Theta(\vec{x}, t, p, \hat{p})$ . Here we try to solve Eq. (48) in general. To do this, we introduce the Fourier transform of the temperature contrast function:

$$\Theta(\vec{x}, t, p, \hat{p}) = \int \frac{d^3 k}{(2\pi)^3} e^{i \vec{k} \cdot \vec{x}} \Theta(\vec{k}, t, p, \hat{p}) . \quad (49)$$

Taking into account that the collision integral depends linearly on  $\Theta$ , we easily obtain:

$$\frac{\partial \Theta(\vec{k}, t, p, \hat{p})}{\partial t} + \frac{i \vec{k} \cdot \hat{p}}{a(t)} \Theta(\vec{k}, t, p, \hat{p}) + \frac{1}{2} \hat{p}^i \hat{p}^j \frac{\partial h_{ij}}{\partial t} \simeq \frac{1}{f_0} \left( \frac{\partial f}{\partial t} \right)_{coll} [\Theta(\vec{k}, t, p, \hat{p})] . \quad (50)$$

To determine the polarization of the cosmic microwave background we need the polarized distribution function which, in general, is represented by a column vector whose components are the four Stokes parameters (Chandrasekhar 1960). In fact, due to the axial symmetry of the metric only two Stokes parameters need to be considered, namely the two intensities of radiation with electric vectors in the plane containing  $\vec{p}$  and  $\vec{n}$  and perpendicular to this plane respectively. As a consequence, instead of Eq. (39) we have:

$$f(\vec{x}, t, p, \hat{p}) \simeq f_0(p, t) \left[ \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \Theta(\vec{x}, t, p, \hat{p}) \right] , \quad (51)$$

where  $\Theta(\vec{x}, t, p, \hat{p})$  is a two component column vector. Using Eq. (3) and defining

$$\mu = \cos \theta_{\vec{p}\vec{n}} , \quad \cos \theta_{\vec{k}\vec{p}} = \frac{\vec{k} \cdot \hat{p}}{k} , \quad (52)$$

we get from Eq. (50):

$$\begin{aligned} \frac{\partial \Theta(\vec{k}, t, \mu)}{\partial t} + \frac{i k}{a(t)} \cos \theta_{\vec{k}\vec{p}} \Theta(\vec{k}, t, \mu) &\simeq \frac{1}{2} \left[ \frac{d}{dt} e^2(t) \right] \mu^2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ - \sigma_T n_e \left[ \Theta(\vec{k}, t, \mu) - \frac{3}{8} \int_{-1}^1 \begin{pmatrix} 2(1-\mu^2)(1-\mu'^2) + \mu^2 \mu'^2 & \mu^2 \\ \mu'^2 & 1 \end{pmatrix} \Theta(\vec{k}, t, \mu') d\mu' \right] \end{aligned} \quad (53)$$

where  $\sigma_T$  is the Thomson cross section and  $n_e(t)$  the electron number density (Chandrasekhar 1960).

Introducing the conformal time:

$$\eta(t) = \int_0^t \frac{dt'}{a(t')} , \quad (54)$$

we rewrite Eq. (53) as:

$$\begin{aligned} \frac{\partial \Theta(\vec{k}, \eta, \mu)}{\partial \eta} + i k \cos \theta_{\vec{k}\vec{p}} \Theta(\vec{k}, \eta, \mu) &\simeq \frac{1}{2} \left[ \frac{d}{d\eta} e^2(\eta) \right] \left( \mu^2 - \frac{1}{3} \right) \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ - a(\eta) \sigma_T n_e \left[ \Theta(\vec{k}, \eta, \mu) - \frac{3}{8} \int_{-1}^1 \begin{pmatrix} 2(1-\mu^2)(1-\mu'^2) + \mu^2 \mu'^2 & \mu^2 \\ \mu'^2 & 1 \end{pmatrix} \Theta(\vec{k}, \eta, \mu') d\mu' \right] \end{aligned} \quad (55)$$

with a suitable overall normalization of the blackbody intensity. To determine the general solutions of Eq. (55) we write (Basko & Polnarev 1980; Cea 2010):

$$\Theta(\vec{k}, \eta, \mu) = \theta_a(\vec{k}, \eta) \left( \mu^2 - \frac{1}{3} \right) \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \theta_p(\vec{k}, \eta) (1 - \mu^2) \begin{pmatrix} 1 \\ -1 \end{pmatrix} . \quad (56)$$

From Eq. (56) it is evident that  $\theta_a$  measures the degree of anisotropy, while  $\theta_p$  gives the polarization of the primordial radiation. With the aid of Eq. (56) we rewrite Eq. (55) as:

$$\begin{aligned} \frac{\partial \theta_a(\vec{k}, \eta)}{\partial \eta} + i k \cos \theta_{\vec{k}\vec{p}} \theta_a(\vec{k}, \eta) &\simeq \Delta H(\eta) - a(\eta) \sigma_T n_e \left[ \frac{9}{10} \theta_a(\vec{k}, \eta) + \frac{3}{5} \theta_p(\vec{k}, \eta) \right] \\ \frac{\partial \theta_p(\vec{k}, \eta)}{\partial \eta} + i k \cos \theta_{\vec{k}\vec{p}} \theta_p(\vec{k}, \eta) &\simeq -a(\eta) \sigma_T n_e \left[ \frac{1}{10} \theta_a(\vec{k}, \eta) + \frac{2}{5} \theta_p(\vec{k}, \eta) \right] \end{aligned} \quad (57)$$

where we introduced the cosmic shear (Negroponte & Silk 1980; Cea 2010):

$$\Delta H(\eta) \equiv \frac{1}{2} \frac{d}{d\eta} e^2(\eta) . \quad (58)$$

The solution of the linear differential system Eq. (57) is the sum of the general solution of the homogeneous system and a particular solution. The solution of the homogeneous system (i.e.  $\Delta H(\eta) = 0$ ) is:

$$\theta_a(\vec{k}, \eta) = \theta_p(\vec{k}, \eta) = 0 , \quad (59)$$

for the are no anisotropies without cosmological perturbations. To determine the particular solution of Eq. (57), we note that the linear combination:

$$\bar{\theta}(\vec{k}, \eta) \equiv \theta_a(\vec{k}, \eta) + \theta_p(\vec{k}, \eta) \quad (60)$$

satisfies the following equation:

$$\frac{\partial \bar{\theta}(\vec{k}, \eta)}{\partial \eta} + i k \cos \theta_{\vec{k}\vec{p}} \bar{\theta}(\vec{k}, \eta) \simeq \Delta H(\eta) - a(\eta) \sigma_T n_e \bar{\theta}(\vec{k}, \eta) . \quad (61)$$

Introducing the optical depth:

$$\tau(\eta, \eta') = \int_{\eta'}^{\eta} \sigma_T n_e a(\eta'') d\eta'' , \quad (62)$$

it is easy to verify that the solution of Eq. (61) is given by:

$$\bar{\theta}(\vec{k}, \eta) = \int_{\eta_i}^{\eta} \Delta H(\eta') e^{-\tau(\eta, \eta')} e^{i k \cos \theta_{\vec{k}\vec{p}}(\eta' - \eta)} d\eta' , \quad (63)$$

where  $\eta_i$  is an early conformal time such that  $\bar{\theta}(\vec{k}, \eta_i) = 0$ . It is now easy to determine  $\theta_a$  and  $\theta_p$ . We get:

$$\theta_a(\vec{k}, \eta) = \frac{1}{7} \int_{\eta_i}^{\eta} \Delta H(\eta') \left[ 6e^{-\tau(\eta, \eta')} + e^{-\frac{3}{10}\tau(\eta, \eta')} \right] e^{i k \cos \theta_{\vec{k}\vec{p}}(\eta' - \eta)} d\eta' , \quad (64)$$

$$\theta_p(\vec{k}, \eta) = \frac{1}{7} \int_{\eta_i}^{\eta} \Delta H(\eta') \left[ e^{-\tau(\eta, \eta')} - e^{-\frac{3}{10}\tau(\eta, \eta')} \right] e^{i k \cos \theta_{\vec{k}\vec{p}}(\eta' - \eta)} d\eta' . \quad (65)$$

In summary, we have found that the temperature fluctuations induced by the spatial anisotropy of the geometry of the universe at large scales is given by Eqs. (56), (64) and (65). Obviously, we are interested in the temperature anisotropies for  $\eta = \eta_0$  ( $\eta_0$  is the conformal time now). As will be evident later on, the main contributions to the integrals in Eqs. (64) and (65) come from conformal times near the decoupling conformal time  $\eta_d$ . Moreover, observing that  $\eta_d \ll \eta_0$  we may write:

$$\Theta(\vec{k}, \eta_0, \mu, \hat{p}) \simeq \theta_a(\mu^2 - \frac{1}{3}) e^{-i k \cos \theta_{\vec{k}\vec{p}} \eta_0} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \theta_p(1 - \mu^2) e^{-i k \cos \theta_{\vec{k}\vec{p}} \eta_0} \begin{pmatrix} 1 \\ -1 \end{pmatrix} , \quad (66)$$

where:

$$\theta_a \simeq \frac{1}{7} \int_{\eta_i}^{\eta_0} \Delta H(\eta') \left[ 6e^{-\tau(\eta_0, \eta')} + e^{-\frac{3}{10}\tau(\eta_0, \eta')} \right] d\eta' , \quad (67)$$

$$\theta_p \simeq \frac{1}{7} \int_{\eta_i}^{\eta_0} \Delta H(\eta') \left[ e^{-\tau(\eta_0, \eta')} - e^{-\frac{3}{10}\tau(\eta_0, \eta')} \right] d\eta' . \quad (68)$$

In appendix A we evaluate the two parameters  $\theta_a$  and  $\theta_p$ . We find (see Eqs. (A15) and (A8) ):

$$\theta_a \simeq -\frac{1}{2} \times 0.944 e_{\text{dec}}^2 , \quad (69)$$

$$\theta_p \simeq 8.92 \cdot 10^{-3} e_{\text{dec}}^2 . \quad (70)$$

#### 4 THE QUADRUPOLE ANOMALY

We are, now, in position to discuss the low quadrupole anomaly in the CMB temperature anisotropies detected by WMAP and recently confirmed by Planck. The temperature anisotropies of the cosmic background depend on the polar angle  $\theta, \phi$ , so that one usually expands in terms of spherical harmonics:

$$\frac{\Delta T(\theta, \phi)}{T_0} = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{+\ell} a_{\ell m} Y_{\ell m}(\theta, \phi), \quad (71)$$

where  $T_0 \simeq 2.7255 \text{ K}$  (Fixsen 2009) is the actual (average) temperature of the CMB radiation. Note that the  $a_{\ell m}$ 's in Eq. (71) are dimensionless and are obtained from the corresponding coefficients in Eq. (5) by dividing by  $T_0$ . After that, one introduces the power spectrum:

$$\left(\frac{\Delta T_{\ell}}{T_0}\right)^2 = \frac{1}{2\pi} \frac{\ell(\ell+1)}{2\ell+1} \sum_m |a_{\ell m}|^2, \quad (72)$$

that fully characterizes the properties of the CMB temperature anisotropy. In particular, we focus on the quadrupole anisotropy  $\ell = 2$ :

$$\mathcal{Q}^2 \equiv \left(\frac{\Delta T_2}{T_0}\right)^2. \quad (73)$$

In the standard model the CMB temperature fluctuations are induced by the cosmological perturbations of the FRW homogeneous and isotropic background metric generated by the inflation-produced potentials. In the ellipsoidal universe we must also consider the effects on the CMB anisotropies induced by the anisotropic expansion of the universe. In fact, as discussed in sec. 2, at large scales the observed anisotropies in the CMB temperature are due to the linear superposition of the two contributions according to Eq. (47). Therefore, we may write:

$$a_{\ell m} = a_{\ell m}^A + a_{\ell m}^I. \quad (74)$$

In the previous section we have determined the contributions to temperature contrast function induced by the anisotropic expansion of the universe:

$$\Theta^A(\vec{k}, \eta_0, \mu, \hat{p}) \simeq \theta_a \left(\mu^2 - \frac{1}{3}\right) e^{-i k \cos \theta_{\vec{k}\vec{p}} \eta_0}, \quad \mu = \cos \theta_{\vec{p}\vec{n}}. \quad (75)$$

In appendix B, starting from Eq. (75) we perform the multipole expansion of the temperature fluctuation correlations and obtain the multipole coefficients  $a_{\ell m}^A$ . However, it is evident from Eq. (75) that the main contribution to the temperature fluctuations is for  $k \simeq 0$ . It is easy to see that this corresponds to solve the Boltzmann equation Eq. (48) by neglecting the spatial dependence on the temperature contrast function. In this case we obtain at once:

$$\frac{\Delta T^A(\theta, \phi)}{T_0} \simeq \theta_a (\cos^2 \theta_{\vec{p}\vec{n}} - \frac{1}{3}), \quad (76)$$

where  $\theta_a$  is given by Eq. (69) and  $\theta, \phi$  are the polar angles of the photon momentum  $\vec{p}$ .

Let  $\theta_n, \phi_n$  be the polar angles of the direction of the axis of symmetry  $\vec{n}$ , then:

$$\frac{\Delta T^A(\theta, \phi)}{T_0} \simeq \frac{2}{3} \theta_a P_2(\cos \theta_{\vec{p}\vec{n}}) = \frac{2}{3} \theta_a \frac{4\pi}{5} \sum_{m=-2}^{+2} Y_{2m}(\theta, \phi) Y_{2m}^*(\theta_n, \phi_n). \quad (77)$$

Since from Eq. (71) it follows that:

$$a_{\ell m}^A = \int d\Omega \frac{\Delta T^A(\theta, \phi)}{T_0} Y_{\ell m}^*(\theta, \phi), \quad (78)$$

we obtain immediately:

$$a_{2m}^A \simeq -\frac{4\pi}{15} \epsilon^2 Y_{2m}^*(\theta_n, \phi_n), \quad \epsilon^2 \equiv 0.944 e_{\text{dec}}^2, \quad (79)$$

while  $a_{\ell m}^A = 0$  for  $\ell \neq 2$ . In other words, at large scales the anisotropy of the metric contributes mainly to the quadrupole CMB temperature anisotropies. From Eq. (79) we find:

$$\begin{aligned} a_{20}^A &\simeq -\frac{4\pi}{15} \epsilon^2 \sqrt{\frac{5}{16\pi}} [1 - 3 \cos^2 \theta_n], \\ a_{21}^A &= (a_{2,-1}^A)^* \simeq +i \frac{4\pi}{15} \epsilon^2 \sqrt{\frac{15}{8\pi}} e^{-i\phi_n} \sin \theta_n \cos \theta_n, \\ a_{22}^A &= (a_{2,-2}^A)^* \simeq \frac{4\pi}{15} \epsilon^2 \sqrt{\frac{15}{32\pi}} e^{-2i\phi_n} \sin^2 \theta_n. \end{aligned} \quad (80)$$



After a little algebra we rewrite Eq. (80) as:

$$\begin{aligned} a_{20}^A &\simeq +\frac{1}{6}\epsilon^2\sqrt{\frac{\pi}{5}}[1+3\cos^2(2\theta_n)] , \\ a_{21}^A &= (a_{2,-1}^A)^* \simeq +i\sqrt{\frac{\pi}{30}}\epsilon^2 e^{-i\phi_n} \sin(2\theta_n) , \\ a_{22}^A &= (a_{2,-2}^A)^* \simeq +\sqrt{\frac{\pi}{30}}\epsilon^2 e^{-2i\phi_n} \sin^2 \theta_n . \end{aligned} \quad (81)$$

Defining the quadrupole anisotropy:

$$\mathcal{Q}_A^2 \equiv \left(\frac{\Delta T_2^A}{T_0}\right)^2 , \quad (82)$$

we find:

$$\mathcal{Q}_A \simeq \frac{2}{5\sqrt{3}}\epsilon^2 . \quad (83)$$

To determine the coefficients  $a_{\ell m}$ , Eq. (74), we need to know the  $a_{\ell m}^I$ 's. First we observe that the temperature anisotropies are real functions, so that we must have  $a_{\ell,-m} = (-1)^m (a_{\ell,m})^*$ . Observing that  $a_{\ell,-m}^A = (-1)^m (a_{\ell,m}^A)^*$  (see Eq. (81)), we have the constraints  $a_{\ell,-m}^I = (-1)^m (a_{\ell,m}^I)^*$ . Moreover, because the standard inflation-produced temperature fluctuations are statistically isotropic, we reasonably assume that the  $a_{2m}^I$  coefficients are equals up to a phase factor. Therefore, we can write:

$$\begin{aligned} a_{20}^I &\simeq \sqrt{\frac{\pi}{3}} \mathcal{Q}_I , \\ a_{21}^I &= -(a_{2,-1}^I)^* \simeq +i\sqrt{\frac{\pi}{3}} e^{i\phi_1} \mathcal{Q}_I , \\ a_{22}^I &= (a_{2,-2}^I)^* \simeq \sqrt{\frac{\pi}{3}} e^{i\phi_2} \mathcal{Q}_I , \end{aligned} \quad (84)$$

where  $0 \leq \phi_1, \phi_2 \leq 2\pi$  are unknown phases. It is easy to check that:

$$\mathcal{Q}_I^2 = \left(\frac{\Delta T_2^I}{T_0}\right)^2 . \quad (85)$$

Using Eq. (7) we obtain the estimate:

$$\mathcal{Q}_I \simeq (12.44 \pm 3.93) 10^{-6} . \quad (86)$$

Taking into account Eqs. (73), (74), (81) and (84) we get for the total quadrupole:

$$\mathcal{Q}^2 = \mathcal{Q}_A^2 + \mathcal{Q}_I^2 + 2f(\theta_n, \phi_n, \phi_1, \phi_2) \mathcal{Q}_A \mathcal{Q}_I , \quad (87)$$

where

$$f(\theta_n, \phi_n, \phi_1, \phi_2) = \frac{1}{4\sqrt{5}}[1+3\cos(2\theta_n)] + \sqrt{\frac{3}{10}}\sin(2\theta_n)\cos(\phi_1+\phi_n) + \sqrt{\frac{3}{10}}\sin^2\theta_n\cos(\phi_2+2\phi_n) . \quad (88)$$

Eqs. (87) and (88) show that, indeed, if the space-time background metric is not isotropic, the quadrupole anisotropy may become smaller than the one expected in the standard isotropic  $\Lambda$ CDM cosmological model of temperature fluctuations. In fact, from Eq. (74) and using Eqs. (81) and (84), we get:

$$a_{20} \simeq +\sqrt{\frac{\pi}{3}} \mathcal{Q}_I + \frac{1}{6}\epsilon^2\sqrt{\frac{\pi}{5}}[1+3\cos^2(2\theta_n)] , \quad (89)$$

$$a_{21} = +i\sqrt{\frac{\pi}{3}} e^{i\phi_1} \mathcal{Q}_I + i\sqrt{\frac{\pi}{30}}\epsilon^2 e^{-i\phi_n} \sin(2\theta_n) , \quad (90)$$

$$a_{22} \simeq +\sqrt{\frac{\pi}{3}} e^{i\phi_2} \mathcal{Q}_I + \sqrt{\frac{\pi}{30}}\epsilon^2 e^{-2i\phi_n} \sin^2 \theta_n . \quad (91)$$

Eqs. (89), (90) and (91) give a system of five equations which can be solved to get the five unknown parameters  $e_{\text{dec}}^2, \theta_n, \phi_n, \phi_1, \phi_2$ . To do this, however, we need the observed values of the  $a_{\ell m}$ 's. In fact, Campanelli, Cea & Tedesco (2007) used the cleaned CMB temperature fluctuation maps of the WMAP data obtained using the internal linear combination with galactic foreground subtraction. In particular, these authors used three different maps (Hinshaw et al. 2007; de Oliveira-Costa & Tegmark 2006; Park, Park & Gott 2007). Actually, the same procedure can be applied to the foreground-cleaned CMB maps obtained from the Planck data as detailed in Ade et al. (2013b). However, irrespective from the adopted CMB cleaned map the quadrupole anomalies detected by WMAP and confirmed by Planck are accounted for if:

$$a_{21} \approx 0, \quad (92)$$

and

$$|a_{20}|^2 \ll 2|a_{22}|^2. \quad (93)$$

In fact, it is easy to check that these equations imply both the almost planarity and the suppression of power of the quadrupole moment. Remarkably, it turns out that Eqs. (92) and (93) allow us to determine the eccentricity at decoupling and constraint the polar angles of the symmetry axis. Inserting Eq. (92) into Eq. (90) we readily obtain:

$$\epsilon^2 \simeq \frac{\sqrt{10} Q_I}{|\sin(2\theta_n)|}, \quad (94)$$

where  $\phi_n + \phi_1 \simeq 0^\circ, 360^\circ$  if  $\sin(2\theta_n) < 0$ , or  $\phi_n + \phi_1 \simeq 180^\circ, 540^\circ$  if  $\sin(2\theta_n) > 0$ . Moreover, from Eq. (89) and taking into account Eq. (93) we find:

$$\cos(2\theta_n) \simeq -\frac{1}{3} - 2\sqrt{\frac{5}{3}} \frac{Q_I}{\epsilon^2}. \quad (95)$$

Combining Eqs. (94) and (95) we obtain:

$$\theta_n \simeq \arctan\left(\pm \frac{\sqrt{6}}{2} + 2\right) \simeq 73^\circ, 107^\circ. \quad (96)$$

This last equation together with Eqs. (86) and Eq. (94) gives the eccentricity at decoupling:

$$e_{\text{dec}} \simeq (0.86 \pm 0.14) 10^{-2}. \quad (97)$$

Finally, using Eqs. (73), (92) and (93) we get:

$$Q^2 \simeq \frac{6}{5\pi} |a_{22}|^2 \simeq \frac{6}{25\pi} Q_I^2 \left[ 1 + \frac{\sin^4 \theta_n}{\sin^2(2\theta_n)} + \frac{2 \sin^2 \theta_n}{|\sin(2\theta_n)|} \cos(\phi_2 + 2\phi_n) \right]. \quad (98)$$

Using the observed value of the quadrupole temperature anisotropy Eq. (6) we estimate from Eq. (98) :

$$\cos(\phi_2 + 2\phi_n) \simeq -0.92 \pm 0.12. \quad (99)$$

To summarize, our almost model independent analysis allowed to fix the eccentricity at decoupling, Eq. (97). As concern the symmetry axis, using the galactic coordinates  $b_n, l_n$ , we found:

$$b_n \simeq \pm 17^\circ, \quad (100)$$

while the longitude  $l_n$  turned out to be poorly constrained in qualitative agreement with Campanelli, Cea & Tedesco (2007).

## 5 THE LARGE SCALE POLARIZATION

In this section we discuss the large scale polarization in the primordial cosmic background. In our previous work (Cea 2010) we argued that the ellipsoidal geometry of the universe induces sizable polarization signal at large scale without invoking the CMB reionization mechanism. If we assume that early CMB reionization is negligible, then it is well known that at large scale the primordial inflation induced cosmological perturbations do not produce sizable polarization signal (Dodelson 2003; Mukhanov 2005). In this case the polarization of the temperature fluctuations are fully given by the anisotropic expansion of the universe. According to our discussion in sect. 3 we may write:

$$\Theta^E(\vec{k}, \eta_0, \mu, \hat{p}) \simeq \theta_p (1 - \cos^2 \theta_{\vec{p}\vec{n}}) e^{-i k \cos \theta_{\vec{k}\vec{p}} \eta_0}, \quad (101)$$

where the superscript E indicates that the temperature polarization contributes only to the so-called E-modes. In fact, Eq. (66) shows that the anisotropy of the metric of the universe gives rise only to a linear polarization of the cosmic background radiation. In appendix B we discuss the multipole expansion of the temperature polarization correlations. As we have already observed, the main contribution to the polarization temperature contrast functions is for  $k \simeq 0$ , which corresponds to neglect the spatial dependence of the solutions of the Boltzmann equation. Thus, we have:

$$\frac{\Delta T^E(\theta, \phi)}{T_0} \simeq \theta_p (1 - \cos^2 \theta_{\vec{p}\vec{n}}) = \frac{2}{3} \theta_p - \frac{2}{3} \theta_p P_2(\cos \theta_{\vec{p}\vec{n}}). \quad (102)$$

We may, now, expand in terms of spherical harmonics as in Eq. (71). It is evident from Eq. (102) that the non-zero multipole coefficients  $a_{\ell m}^E$  are for the monopole  $\ell = 0$  and the quadrupole  $\ell = 2$ . The monopole term determines the average large scale polarization of the cosmic microwave background:

$$\frac{\Delta T_{\text{pol}}}{T_0} \equiv \frac{1}{4\pi} \int d\Omega \frac{\Delta T^E(\theta, \phi)}{T_0} \simeq \frac{2}{3} \theta_p. \quad (103)$$

Using Eqs. (70) and (97) we obtain:

$$\Delta T_{pol} \simeq (1.20 \pm 0.38) \mu K , \quad (104)$$

in qualitative agreement with our previous estimate (Cea 2010).

On the other hand, from Eq. (102) we easily obtain:

$$a_{2m}^E \simeq -\frac{8\pi}{15} \theta_p Y_{2m}^*(\theta_n, \phi_n) , \quad (105)$$

which implies:

$$\begin{aligned} a_{20}^E &\simeq -\frac{8\pi}{15} \theta_p \sqrt{\frac{5}{16\pi}} [1 - 3 \cos^2 \theta_n] , \\ a_{21}^E &= (a_{2,-1}^A)^* \simeq +i \frac{8\pi}{15} \theta_p \sqrt{\frac{15}{8\pi}} e^{-i\phi_n} \sin \theta_n \cos \theta_n , \\ a_{22}^E &= (a_{2,-2}^A)^* \simeq \frac{8\pi}{15} \theta_p \sqrt{\frac{15}{32\pi}} e^{-2i\phi_n} \sin^2 \theta_n , \end{aligned} \quad (106)$$

or better:

$$\begin{aligned} a_{20}^E &\simeq +\frac{1}{3} \theta_p \sqrt{\frac{\pi}{5}} [1 + 3 \cos^2(2\theta_n)] , \\ a_{21}^E &= (a_{2,-1}^A)^* \simeq +2i \sqrt{\frac{\pi}{30}} \theta_p e^{-i\phi_n} \sin(2\theta_n) , \\ a_{22}^E &= (a_{2,-2}^A)^* \simeq +2 \sqrt{\frac{\pi}{30}} \theta_p e^{-2i\phi_n} \sin^2 \theta_n . \end{aligned} \quad (107)$$

Eq. (107) allows to evaluate the quadrupole EE correlation:

$$\left(\frac{\Delta T_2^{EE}}{T_0}\right)^2 = \frac{3}{5\pi} \sum_{m=-2}^{m=+2} |a_{2m}^E|^2 . \quad (108)$$

In fact, a straightforward calculation gives:

$$\left(\frac{\Delta T_2^{EE}}{T_0}\right)^2 \simeq \frac{16}{75} \theta_p^2 . \quad (109)$$

Using again Eqs. (70) and (97) we find:

$$\Delta T_2^{EE} \simeq 0.83 \pm 0.27 \mu K . \quad (110)$$

We may, also, estimate the quadrupole TE correlation:

$$\left(\frac{\Delta T_2^{TE}}{T_0}\right)^2 = \frac{3}{5\pi} \sum_{m=-2}^{m=+2} a_{2m}^T (a_{2m}^E)^* = \frac{3}{5\pi} \left\{ a_{20}^T a_{20}^E + 2 \operatorname{Re}[a_{21}^T (a_{21}^E)^*] + 2 \operatorname{Re}[a_{22}^T (a_{22}^E)^*] \right\} , \quad (111)$$

where the  $a_{2m}^T$ 's are given by Eqs. (89) - (91). Using Eqs. (92) and (93), we may simplify Eq. (111) as:

$$\left(\frac{\Delta T_2^{TE}}{T_0}\right)^2 \simeq \frac{6}{5\pi} \operatorname{Re}[a_{22}^T (a_{22}^E)^*] \simeq \frac{4}{5\sqrt{10}} \theta_p \sin^2 \theta_n \left[ \mathcal{Q}_I \cos(\phi_2 + 2\phi_n) + \frac{1}{\sqrt{10}} \epsilon^2 \sin^2 \theta_n \right] . \quad (112)$$

After using Eq. (94), this last equation can be rewritten as:

$$\left(\frac{\Delta T_2^{TE}}{T_0}\right)^2 \simeq \frac{4}{5\sqrt{10}} \theta_p \mathcal{Q}_I \sin^2 \theta_n \left[ \cos(\phi_2 + 2\phi_n) + \frac{\sin^2 \theta_n}{|\sin(2\theta_n)|} \right] . \quad (113)$$

Finally, using our previous estimates Eqs. (96) and (99) we find:

$$\Delta T_2^{TE} \simeq 3.14 \pm 0.76 \mu K . \quad (114)$$

## 6 CONCLUSIONS

In this paper we solved at large scales the Boltzmann equation for the CMB photon distribution function by considering the effects of the inflation primordial scalar perturbations and the anisotropy of the geometry in the ellipsoidal universe model. We showed explicitly that the CMB temperature fluctuations are obtained by the linear superimposition of the temperature fluctuations induced by the cosmological scalar perturbations and by the spatial anisotropy of the metric. We found that the anisotropic expansion of the universe, the so-called cosmic shear, affects mainly the quadrupole correlation functions.

Moreover, we showed that these effects extend also to the low-lying multipoles  $\ell \sim 10$ .

We confirmed previous results that the low quadrupole temperature correlation, detected by WMAP and by the Planck satellite, could be accounted for if the geometry of the universe is plane-symmetric with eccentricity at decoupling of order  $10^{-2}$ . We showed that the ellipsoidal geometry of the universe produces sizable polarization signal at large scales. We found that our estimate of the quadrupole TE correlation were in agreement both in sign and magnitude with observations. On the other hand, regarding the quadrupole EE correlation our result did not compare well with the final analysis of the Wilkinson Microwave Anisotropy Probe collaboration. However, we feel that the rather low polarization signal detected by WMAP at large scales could be due to an overestimation of the foreground polarization signal. In fact, in the standard reionization scenario the large scale polarization in the temperature fluctuations is produced by the fraction of the rescattered photons on the scales corresponding to the reionization horizon. As a consequence in this usually adopted scenario the polarization anisotropies are present for  $\ell \geq 2$ . That means, in particular, that there is no average polarization. On the other hand, we have shown that the anisotropic expansion in the ellipsoidal universe model implies the presence of large scale polarization in the temperature fluctuations without invoking reionization processes. In fact, at variance with the usually accepted scenario, in the ellipsoidal universe we have a sizeable average polarization signal at level  $\sim \mu K$ . If this average polarization in the temperature fluctuations of the cosmic background is misinterpreted as foreground polarization signal, then it could result in a considerable underestimate of the CMB polarization signal at large scales. Therefore a careful characterization of foreground polarization is certainly crucial for polarization measurements.

In conclusion, we are reinforcing the proposal that the ellipsoidal universe cosmological model is a viable alternative that could account for the detected large scale anomalies in the cosmic microwave anisotropies.

## APPENDIX A: EVALUATION OF THE PARAMETERS $\theta_A$ AND $\theta_P$

In this Appendix we evaluate the parameters  $\theta_a$  and  $\theta_p$  given by Eqs. (67) and (68). In fact, these two parameters have been estimate in Cea (2010) by assuming that the plane-symmetric geometry is induced by a cosmological magnetic field. Presently, we would like to present a slightly better estimate which is valid irrespective of the physical mechanism responsible for the generation of the spatial anisotropy in the early universe.

Let us consider, firstly, the parameter  $\theta_p$ . Defining  $\tau(\eta) = \tau(\eta_0, \eta)$ , it is easy to verify that  $\tau(\eta', \eta) = \tau(\eta') - \tau(\eta)$ . Observing that  $\tau(\eta_0) \simeq 0$ , we rewrite Eq. (68) as:

$$\theta_p \simeq \frac{1}{7} \int_{\eta_i}^{\eta_0} \Delta H(\eta') \left[ e^{-\tau(\eta')} - e^{-\frac{3}{10}\tau(\eta')} \right] d\eta' . \quad (\text{A1})$$

It is convenient to rewrite the integral in Eq. (A1) in terms of the cosmic time:

$$\theta_p \simeq \frac{1}{7} \int_{t_i}^{t_0} \frac{1}{2} \frac{d}{dt'} e^2(t') \left[ e^{-\tau(t')} - e^{-\frac{3}{10}\tau(t')} \right] dt' , \quad (\text{A2})$$

where  $t_0$  is the age of the universe and we used Eq. (58). To evaluate the derivative in Eq. (A2), we note that in general (Campanelli, Cea & Tedesco 2007)  $e^2(t) \sim a(t)^{-\frac{3}{2}}$ . Thus, in the matter-dominated era we may write near decoupling:

$$\frac{1}{2} \frac{d}{dt} e^2(t) \simeq -\frac{3}{4} e^2(t) H(t) . \quad (\text{A3})$$

After changing the integration variable by using instead of the cosmic time  $t$  the red-shift  $z$ , we obtain:

$$\theta_p \simeq -\frac{3}{28} \int_0^\infty \frac{e^2(z')}{1+z'} \left[ e^{-\tau(z')} - e^{-\frac{3}{10}\tau(z')} \right] dz' . \quad (\text{A4})$$

Since near decoupling we may write:

$$e^2(z) \simeq e_{\text{dec}}^2 \left( \frac{1+z}{1+z_d} \right)^{\frac{3}{2}} , \quad e_{\text{dec}}^2 = e^2(z_d) , \quad (\text{A5})$$

where  $z_d \simeq 1090$  is the red-shift at decoupling, we get:

$$\theta_p \simeq -\frac{3}{28} e_{\text{dec}}^2 \int_0^\infty \left( \frac{1+z}{1+z_d} \right)^{\frac{3}{2}} \frac{1}{1+z} \left[ e^{-\tau(z)} - e^{-\frac{3}{10}\tau(z)} \right] dz . \quad (\text{A6})$$

Finally, it is known that near decoupling to a good approximation one can write (Jones & Wyse 1985):

$$\tau(z) \simeq 0.37 \left( \frac{z}{1000} \right)^{14.25} , \quad 500 \lesssim z \lesssim 1400 . \quad (\text{A7})$$

This allow us to evaluate numerically the integral in Eq. (A6). We obtain:

$$\theta_p \simeq 8.92 \cdot 10^{-3} e_{\text{dec}}^2 . \quad (\text{A8})$$

To evaluate the parameter  $\theta_a$ , we note that:

$$\theta_a \simeq \frac{1}{7} \int_{\eta_i}^{\eta^*} \Delta H(\eta') \left[ 6e^{-\tau(\eta')} + e^{-\frac{3}{10}\tau(\eta')} \right] d\eta' + \int_{\eta^*}^{\eta_0} \Delta H(\eta') d\eta', \quad (\text{A9})$$

where  $\eta^*$  is a conformal time such that  $\tau(\eta) = 0$  for  $\eta \geq \eta^*$ . The second integral in the right hand in Eq. (A9) is elementary:

$$\int_{\eta^*}^{\eta_0} \Delta H(\eta') d\eta' = \int_{\eta^*}^{\eta_0} \frac{1}{2} \frac{d}{d\eta'} e^2(\eta') d\eta' = \int_{t^*}^{t_0} \frac{1}{2} \frac{d}{dt'} e^2(t') dt' = \frac{1}{2} e^2(t_0) - \frac{1}{2} e^2(t^*) = -\frac{1}{2} e^2(t^*), \quad (\text{A10})$$

since  $e^2(t_0) = 0$ . On the other hand, using Eq. (A3) we have:

$$\frac{1}{7} \int_{\eta_i}^{\eta^*} \Delta H(\eta') \left[ 6e^{-\tau(\eta')} + e^{-\frac{3}{10}\tau(\eta')} \right] d\eta' \simeq -\frac{3}{4} \int_{t_i}^{t^*} e^2(t') H(t') \left[ \frac{6}{7} e^{-\tau(t')} + \frac{1}{7} e^{-\frac{3}{10}\tau(t')} \right] dt'. \quad (\text{A11})$$

After using Eq. (A5) we obtain:

$$\theta_a \simeq -\frac{1}{2} e_{\text{dec}}^2 f(z^*), \quad (\text{A12})$$

where:

$$f(z^*) = \left( \frac{1+z^*}{1+z_d} \right)^{\frac{3}{2}} + \frac{3}{2} \int_{z^*}^{\infty} \left( \frac{1+z}{1+z_d} \right)^{\frac{3}{2}} \frac{1}{1+z} \left[ \frac{6}{7} e^{-\tau(z)} + \frac{1}{7} e^{-\frac{3}{10}\tau(z)} \right] dz. \quad (\text{A13})$$

The integral in Eq. (A13) can be evaluated numerically. In fact, we find that  $f(z^*)$  is almost independent on  $z^*$ :

$$f(z^*) \simeq 0.944, \quad 200 \leq z^* \leq 900. \quad (\text{A14})$$

Thus, our final result is:

$$\theta_a \simeq -\frac{1}{2} \times 0.944 e_{\text{dec}}^2. \quad (\text{A15})$$

## APPENDIX B: MULTIPOLE EXPANSION OF THE LARGE SCALE TEMPERATURE ANISOTROPIES

In this appendix we would like to discuss the multipole expansion of the temperature fluctuation correlation functions. According to the results in sec. 3 we have (omitting the superscript A):

$$\Theta^T(\vec{k}, \vec{n}, \hat{p}) \simeq \theta_a \left( \cos^2 \theta_{\vec{n}\hat{p}} - \frac{1}{3} \right) e^{-i k \cos \theta_{\vec{k}\hat{p}} \eta_0}, \quad (\text{B1})$$

$$\Theta^E(\vec{k}, \vec{n}, \hat{p}) \simeq \theta_p (1 - \cos^2 \theta_{\vec{n}\hat{p}}) e^{-i k \cos \theta_{\vec{k}\hat{p}} \eta_0}, \quad (\text{B2})$$

corresponding to the temperature and polarization contrast functions, respectively. For definiteness, let us discuss firstly the temperature-temperature correlations. As is well known (Dodelson 2003) we need to evaluate:

$$\langle \Theta^T(\vec{x}, \vec{n}, \hat{p}) \Theta^T(\vec{x}, \vec{n}, \hat{p}') \rangle = \int \frac{d^3 k}{(2\pi)^3} \Theta^T(\vec{k}, \vec{n}, \hat{p}) [\Theta^T(\vec{k}, \vec{n}, \hat{p}')]^*. \quad (\text{B3})$$

After expanding  $\Theta^T(\vec{x}, \vec{n}, \hat{p})$  in spherical harmonics, one obtains:

$$\langle \Theta^T(\vec{x}, \vec{n}, \hat{p}) \Theta^T(\vec{x}, \vec{n}, \hat{p}') \rangle = \sum_{\ell} C_{\ell}, \quad C_{\ell} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{+\ell} C_{\ell m}, \quad (\text{B4})$$

with:

$$C_{\ell m} = \int \frac{d^3 k}{(2\pi)^3} \int d\Omega_{\hat{p}} d\Omega_{\hat{p}'} Y_{\ell m}^*(\hat{p}) \Theta^T(\vec{k}, \vec{n}, \hat{p}) Y_{\ell m}(\hat{p}') [\Theta^T(\vec{k}, \vec{n}, \hat{p}')]^*. \quad (\text{B5})$$

Now we use the well known identities (Abramowitz & Stegun 1970):

$$P_{\ell}(\hat{x} \cdot \hat{x}') = \frac{4\pi}{2\ell+1} \sum_{m=-\ell}^{m=+\ell} Y_{\ell m}^*(\hat{x}) Y_{\ell m}(\hat{x}'), \quad (\text{B6})$$

and

$$e^{-i \vec{k} \cdot \vec{x}} = \sum_{\ell=0}^{+\infty} i^{\ell} (2\ell+1) j_{\ell}(kx) P_{\ell}(\hat{k} \cdot \hat{x}) = 4\pi \sum_{\ell m} i^{\ell} j_{\ell}(kx) Y_{\ell m}(\hat{k}) Y_{\ell m}^*(\hat{x}), \quad (\text{B7})$$

to get:

$$C_{\ell m} = \frac{8}{9\pi} \theta_a^2 \sum_{\ell_1 m_1} \int_0^{\infty} dk k^2 j_{\ell_1}^2(k\eta_0) \int d\Omega_{\hat{p}} Y_{\ell m}^*(\hat{p}) Y_{\ell_1 m}^*(\hat{p}) P_2(\hat{p} \cdot \hat{n}) \int d\Omega_{\hat{p}'} Y_{\ell m}(\hat{p}') Y_{\ell_1 m}(\hat{p}') P_2(\hat{p}' \cdot \hat{n}). \quad (\text{B8})$$

Using again Eq. (B6) we rewrite Eq. (B8) as:

$$C_{\ell m} = \frac{8}{9\pi} \left(\frac{4\pi}{5}\right)^2 \theta_a^2 \sum_{\ell_1 m_1} \sum_{m_2=-2}^{+2} \int_0^\infty dk k^2 j_{\ell_1}^2(k\eta_0) Y_{2m_2}(\hat{n}) Y_{2m_2}^*(\hat{n}) \left| \int d\Omega_{\hat{p}} Y_{\ell m}(\hat{p}) Y_{\ell_1 m_1}(\hat{p}) Y_{2m_2}(\hat{p}) \right|^2. \quad (\text{B9})$$

The angular integral can be expressed in terms of the Wigner 3j symbols (Messiah 1961):

$$\int d\Omega_{\hat{p}} Y_{\ell_1 m_1}(\hat{p}) Y_{\ell_2 m_2}(\hat{p}) Y_{\ell_3 m_3}(\hat{p}) = \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}. \quad (\text{B10})$$

In our case, using the well-known properties of the 3j symbols, we have the constraints:

$$\ell_1 = \ell, \ell \pm 2. \quad (\text{B11})$$

Actually, we are interested in the limit of large  $\ell$ . To this end, we use the estimate for the asymptotic limit of the average 3j symbols (Borodin, Kroshilin & Tolmachev 1978):

$$\left\langle \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}^2 \right\rangle \simeq \frac{1}{2\pi\ell^2}, \quad \ell_1 \simeq \ell, \quad m_1 \simeq m, \quad (\text{B12})$$

to get:

$$C_{\ell m} \simeq \frac{8}{9\pi} \theta_a^2 \frac{5}{4\pi^3} \frac{1}{\ell^2} \int_0^\infty dk k^2 j_\ell^2(k\eta_0) \left(\frac{4\pi}{5}\right)^2 \sum_{m_2=-2}^{+2} Y_{2m_2}(\hat{n}) Y_{2m_2}^*(\hat{n}). \quad (\text{B13})$$

Since

$$\frac{4\pi}{5} \sum_{m_2=-2}^{+2} Y_{2m_2}(\hat{n}) Y_{2m_2}^*(\hat{n}) = P_2(1) = 1, \quad (\text{B14})$$

we obtain:

$$C_\ell = \frac{1}{2\ell+1} \sum_{m=-\ell}^{+\ell} C_{\ell m} \simeq \frac{8}{9\pi^3} \theta_a^2 \frac{1}{\ell^2} \int_0^\infty dk k^2 j_\ell^2(k\eta_0). \quad (\text{B15})$$

To evaluate the integral over  $k$ , we note that:

$$j_\ell(x) = \sqrt{\frac{\pi}{2x}} J_{\ell+\frac{1}{2}}(x). \quad (\text{B16})$$

So that we are left with the following integrals:

$$I_\ell \equiv \int_0^\infty dk k^2 j_\ell^2(k\eta_0) = \frac{\pi}{2} \int_0^\infty dk \frac{k}{\eta_0} J_{\ell+\frac{1}{2}}^2(k\eta_0). \quad (\text{B17})$$

It is easy to see that the integrals in Eq. (B17) are divergent in the ultraviolet region  $k \rightarrow \infty$ . This divergence is an artifact of our approximations. To overcome this problem we must cut-off the spectrum for high wavenumbers. To our purpose it is enough to assume a power law cut-off function  $k^{-\alpha}$ ,  $0 < \alpha < 1$ . Thus we obtain:

$$I_\ell = \frac{\pi}{2\eta_0^{3-\alpha}} \int_0^\infty dx x^{1-\alpha} J_{\ell+\frac{1}{2}}^2(x). \quad (\text{B18})$$

Using (Gradshteyn & Ryzhik 1983):

$$\int_0^\infty dt t^{-\lambda} J_\nu(t) J_\mu(t) = \frac{\Gamma(\lambda) \Gamma(\frac{\mu+\nu-\lambda+1}{2})}{2^\lambda \Gamma(\frac{\mu-\nu+\lambda+1}{2}) \Gamma(\frac{\mu+\nu+\lambda+1}{2}) \Gamma(\frac{-\mu+\nu+\lambda+1}{2})}, \quad (\text{B19})$$

we obtain:

$$I_\ell = \frac{\pi}{2\eta_0^{3-\alpha}} \frac{1}{2^{\alpha-1}} \frac{\Gamma(\alpha-1)}{[\Gamma(\frac{\alpha}{2})]^2} \frac{\Gamma(\ell+\frac{3}{2}-\frac{\alpha}{2})}{\Gamma(\ell+\frac{1}{2}+\frac{\alpha}{2})}. \quad (\text{B20})$$

For large  $\ell$  we use the estimate (Gradshteyn & Ryzhik 1983):

$$\lim_{|z| \rightarrow \infty} \frac{\Gamma(z+a)}{\Gamma(z)} = e^{-a \ln |z|} = |z|^{-a}, \quad (\text{B21})$$

to obtain:

$$I_\ell \sim \frac{\pi}{2^\alpha \eta_0^{3-\alpha}} \frac{\Gamma(\alpha-1)}{[\Gamma(\frac{\alpha}{2})]^2} \ell^{-1+\alpha}. \quad (\text{B22})$$

Inserting Eq. (B22) Eq. (B15) we get:

$$\frac{\ell(\ell+1)}{2\pi} C_\ell \sim \frac{8}{9\pi^2} \theta_a^2 \frac{1}{2^\alpha \eta_0^{3-\alpha}} \frac{\Gamma(\alpha-1)}{[\Gamma(\frac{\alpha}{2})]^2} \ell^{-1+\alpha}, \quad 0 < \alpha < 1. \quad (\text{B23})$$

This last equation shows, indeed, that the anisotropy of the metric contributes mainly at large scales affecting only the low-lying multipoles, at least for the temperature-temperature anisotropy correlations.

For the polarization correlations, we rewrite Eq. (B2) as:

$$\Theta^E(\vec{k}, \vec{n}, \hat{p}) \simeq \frac{2}{3} \theta_p e^{-i k \cos \theta_{\vec{k}\vec{p}} \eta_0} - \frac{2}{3} \theta_p P_2(\cos \theta_{\vec{n}\vec{p}}) e^{-i k \cos \theta_{\vec{k}\vec{p}} \eta_0}. \quad (\text{B24})$$

Therefore, we have two contributions to the EE correlations. The second term on the right hand of Eq. (B24) is analogous to the temperature contrast function Eq. (B1), while the first term would contribute to the coefficient  $C_\ell$  with:

$$C_\ell \simeq \frac{8}{9\pi} \theta_p^2 \int_0^\infty dk k^2 j_\ell^2(k\eta_0). \quad (\text{B25})$$

Using Eqs. (B19) - (B21) we find:

$$\frac{\ell(\ell+1)}{2\pi} C_\ell \sim \frac{8}{9} \theta_p^2 \frac{1}{2^\alpha \eta_0^{3-\alpha}} \frac{\Gamma(\alpha-1)}{[\Gamma(\frac{\alpha}{2})]^2} \ell^{1+\alpha}, \quad 0 < \alpha < 1. \quad (\text{B26})$$

Eq. (B26) would imply that the EE correlation functions due to the anisotropy of the metric are sizable not only for the low-lying multipoles, but also for higher multipoles. However, from Eq. (65) we see that the polarization of the cosmic microwave background (without reionization) at the present time is essentially that produced around the time of recombination, since much later the free electron density is negligible, while much earlier the optical depth is very large. Then, the present polarization is the result of Thomson scattering around the time of decoupling of matter and radiation, which occurs after the free electron density starts to drop significantly (Peebles 1993). Moreover, to obtain Eq. (66) we assumed that  $k \Delta\eta_d \ll 1$ , where  $\Delta\eta_d$  is the conformal time duration of the decoupling process (the thickness of the last scattering surface). In fact, for  $k \Delta\eta_d \gg 1$  the oscillations in the integrand produce a cancellation of the temperature anisotropy polarization. In other words, the finite thickness of the last scattering surface damps the final temperature polarization on these scales. Thus, for wavelengths comparable or smaller than the width of the last scattering surface, the polarization should fall off very rapidly. Indeed, the polarization signal should be confined up to multipoles such that  $\frac{1}{\ell} \sim \frac{\Delta z_d}{z_d} \sim 10^{-1}$ . Actually, more precise statements can be only obtained by solving numerically the radiative transfer equation for the cosmic microwave background including polarization in anisotropic universes. Remarkably, quite recently Pontzen & Challinor (2007) have derived the radiative transfer equation in the nearly Friedmann-Robertson-Walker limit of homogeneous, but anisotropic, universes classified via their Bianchi type. In fact, these authors argued that the polarization signal is mostly confined to multipoles  $\ell \lesssim 10$ .

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